Estimating Monthly GDP in an Exact Kalman Filter Framework

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Abstract

In this project, we describe a framework that nests a great variety of interpolation setups and relaxes the co-integration conditions of temporal disaggregation in the literature. Our goal is to evaluate alternative interpolation models and then to produce a monthly deseasonalized Taiwan's real gross domestic product and make it available for researchers and practitioners. Our empirical result shows that the monthly estimates, incorporated with the information obtained from the related series, are consistent with the quarterly figures. These estimates could be very helpful for short-run policy analysis by signalling any emerging economic problems.

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1 Introduction

A problem often encountered by Council for Economic Planning and Development (CEPD) of the Executive Yuan and more generally by macroeconomic researchers is the interpolation or distribution of economic time series. For interpolation, the task is to estimate the missing values of a stock variable in a higher frequency when the observed values of the series are only reported at less frequent periods. In the distribution case, on which this paper concentrates, the problem concerns the estimation of intraperiod values for a flow variable subjected to the constraint that their sums, or averages, equal the aggregates over the lower frequency of observations. These two processes, usually called “temporal disaggregation techniques”, play an important role for the estimation of short-term economic indicators. Several national statistical institutes of European countries, including France, Italy, Spain, Belgium and Portugal, have made extensive use of these techniques to estimate the Euro area GDP; see e.g., Eurostat (1999) for more details.

The need for temporal disaggregation can stem from a number of reasons. For example, in multiple time-series analysis, monthly data are available for all series but one, for which only quarterly data are observed. Instead of aggregating all other series to quarterly totals which produces specification errors (Rossana and Seater, 1995) and leads to a considerable loss of information, it is more reasonable to disaggregate the quarterly data to monthly figures. For another example, consider the case that a time series has been observed annually over several decades. Due to the increasing importance of the series, the official bureau decided to release quarterly figures for the variables some years ago. Thus, the researchers have a time series with annual observations in the first part and quarterly figures in the remainder. A well-known example is that the annual observations of the Taiwan GDP series have been available since 1951, but the quarterly figures of the series were reported only after 1961. In order to fully utilize all the available data for time series modelling, it is desirable to disaggregate the previous yearly observations to quarterly values.

Temporal disaggregation techniques have been widely studied in the time series literature. Broadly speaking, methods for disaggregation can be classified into two approaches, namely (1) methods that involve the use of observed related series at the desired higher frequency and (2) methods that rely only on purely time series models and do not make use of the information obtained from the related series. The former has been discussed by a number of authors, of whom one of the first was Friedman (1962). Subsequent contributions have been made by Chow and Lin (1971), Denton (1971), Ginsburgh (1973), Gilbert (1977), Fernández (1981), Litterman (1983) and de Alba (1988), to name a few. The latter approach was explored by Wei and Stram (1990) and Guerrero (1990). It depends on the autoregressive integrated moving-average (ARIMA) representation of the series to be disaggregated. Although these two approaches are potentially applicable to a wide variety of cases, they rely mainly on undesired and/or arbitrary assumptions. For example, the former does not accommodate the possibility of some dynamic structure; it also postulates full co-integration relation between the nonstationary related series and the unobserved disaggregated time series *à priori*.[1] The latter extracts signals only from the presumed stochastic process of the series in a way that no other related information is added.

This project concentrates on an alternative method, namely the state-space approach, which was first introduced by Harvey (1989) and later developed by Harvey and Koopman (1997). Given the state-space representation discussed in this project, it can be seen that the proposed model is able to describe dynamic structure of disaggregated time series and allows for high frequency related series without imposing the assumption of co-integration relation. The exact Kalman filtering and smoothing algorithms recently introduced by Koopman (1997) and Koopman and Durbin (2003) are then used to evaluate the

[1] It has been shown that mis-imposing co-integration constraints in the series is dangerous and may result in inferior forecasts; see e.g., Reinsel and Ahn (1992) and Lin and Tsay (1996) for more details.

likelihood function and to estimate real gross domestic product (GDP) at monthly intervals. A major advantage of using the Kalman filter is that it computes the optimal estimates of the latent monthly GDP. It also provides a way to calculate exact finite-sample forecasts based on the appropriate information set. In the empirical study, we apply the proposed model to seasonally adjusted, quarterly Taiwan's real GDP for the period of 1961:I to 2006:II with 182 observations. The smoothing estimate and the associated confidence intervals are thus reported for up-to-date information about the state of Taiwan's economy.

The project is organized as follows: Section 2 introduces the state space methodology and shows how it can be applied to series that can be modelled by nonstationary ARIMA processes. Section 3 presents the empirical analysis of Taiwan's real GDP based on the proposed model. Section 4 presents our conclusions. Appendix briefly reviews the exact Kalman filter and smoother algorithms.

2 State-Space Approach

The basic idea of the state space model is that an observable time series under study can be explained by a vector of unobserved components. The unobserved vector which is assumed to be a first-order Markov process is linked to the observed variable via a measurement equation. Such a modelling approach provides a unified methodology for treating a wide range of problems in time series analysis. Once a model has been put in the state space form, the Kalman filter may be applied and this in turn leads to algorithms for filtering and smoothing. In the present project the aim is to extract unobservable monthly GDP from the published quarterly real GDP, subject to the constraint that the sum of past and current monthly GDP must equal the quarterly data. In addition, the estimation of the latent monthly GDP is based on the presumed ARIMA process and related observed monthly indicators.

2.1 The State-Space Representation

To illustrate our basic idea, let $\ddot{y}_\tau$ be the seasonally adjusted, quarterly real GDP at time τ and $\ddot{\boldsymbol{y}}_\tau = (0\ 0\ \ddot{y}_\tau)'$ be a 3×1 vector of observations. Then we stack the observations $\ddot{\boldsymbol{y}}_\tau$ in one column vector to get $\boldsymbol{y} = (\ddot{\boldsymbol{y}}'_1\ \ddot{\boldsymbol{y}}'_2\ \ldots\ \ddot{\boldsymbol{y}}'_T)'$, where T is the number of quarterly real GDP. Also let y_t denote the t^{th} element of $\boldsymbol{y}$. We assume that the unobserved monthly GDP, y_t^*, satisfies the sum-up constraint

$$y_t = \sum_{i=0}^{2} y_{t-i}^*, \quad t = 3, 6, 9, \ldots, 3T. \tag{1}$$

We further assume that the difference of y_t^* follows an ARMAX(p, q) process

$$\Psi(B) z_t^* = \boldsymbol{x}_t' \boldsymbol{\beta} + \Phi(B)\varepsilon_t, \quad t = 1, 2, 3, \ldots, 3T, \tag{2}$$

where ε_t is a martingale difference sequence with mean zero and variance σ_ε^2, $z_t^* = y_t^* - y_{t-1}^*$, $\Psi(B) = 1 - \psi_1 B - \cdots - \psi_p B^p$ and $\Phi(B) = 1 + \varphi_1 B + \cdots + \varphi_q B^q$ are finite-order polynomials of the back-shift operator B such that they have no common factors and their roots are all outside the unit circle. The related monthly indicators $\boldsymbol{x}_t$ are observable and weakly stationary. As can be seen in (1) and (2), the proposed model is capable of exhibiting the dynamic patterns of monthly GDP and allows for related exogenous variables. Moreover, in our model, it is only assumed that there exists a linear relationship between z_t^* and $\boldsymbol{x}_t$ which are all weakly stationary. Hence, any co-integration restriction between y_t^* and related variables is not required.

It is now easy to show that the proposed model in (1) and (2) can be expressed as a state-space model with the following measurement and transition equations:[2]

$$\begin{aligned} y_t &= \boldsymbol{h}_t' \boldsymbol{\gamma}_t, \\ \boldsymbol{\gamma}_{t+1} &= \boldsymbol{\mu}_t + \boldsymbol{F}\boldsymbol{\gamma}_t + \boldsymbol{R}\varepsilon_{t+1}, \end{aligned} \tag{3}$$

[2] Another alternative not treated in this project to introduce the sum-up constraint is to augment the state-space representation with a "cumulator function", which accumulates monthly GDP observations in a given quarter; see Harvey (1989) and Proietti (2006) for more details.

for $t = 1, 2, 3, \ldots, 3T$, where a $(r+2)$-dimensional vector $\boldsymbol{h}_t = (1\ 0\ \cdots\ 0\ 2\ 1)'$ if $t = 3, 6, 9, \ldots, 3T$ and $\boldsymbol{h}_t = \mathbf{0}$ otherwise;

$$\boldsymbol{\gamma}_t = \begin{bmatrix} z_t^* \\ \sum_{i=2}^r \psi_i z_{t-i+1}^* + \sum_{i=2}^r \varphi_{i-1}\varepsilon_{t-i+2} \\ \sum_{i=3}^r \psi_i z_{t-i+2}^* + \sum_{i=3}^r \varphi_{i-1}\varepsilon_{t-i+3} \\ \vdots \\ \psi_r z_{t-1}^* + \varphi_{r-1}\varepsilon_t \\ y_{t-1}^* \\ y_{t-2}^* \end{bmatrix}_{(r+2)\times 1},$$

$r = \max(p, q+1)$, $\psi_i = 0$ for $i > p$ and $\varphi_i = 0$ for $i > q$. The terms $\boldsymbol{F}$ and $\boldsymbol{R}$ are fixed matrixes such that

$$\boldsymbol{F} = \begin{bmatrix} \psi_1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \psi_2 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_{r-1} & 0 & 0 & \cdots & 1 & 0 & 0 \\ \psi_r & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{(r+2)\times(r+2)}, \quad \boldsymbol{R} = \begin{bmatrix} 1 \\ \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_{r-1} \\ 0 \\ 0 \end{bmatrix}_{(r+2)\times 1},$$

and $\boldsymbol{\mu}_t = (\boldsymbol{x}_{t+1}'\boldsymbol{\beta}\ 0\ \cdots\ 0)'$ is a $(r+2)\times 1$ vector. The state-space representation in (3) is sufficiently rich to accommodate the traditional linear disaggregation techniques. It can encompass the most popular techniques such as Chow and Lin (1971), Hendry and Mizon (1978), Frenández (1981) and Litterman (1983); see Proietti (2006) for more detailed comparisons between these techniques.

2.2 The Exact Initial Kalman Filter

Because $\{y_t\}$ in (3) is nonstationary, the density of the observations does not exist and so the likelihood is not defined in the usual sense. We therefore follow De Jong (1991) and deal with the likelihood evaluation by using a diffuse initial state in the state space model.[3] Instead of estimating y_0^* and y_{-1}^* in the initial state vector (i.e., $\boldsymbol{\gamma}_1$), we treat these elements as diffuse random elements. A simple approximate technique to handle diffuse random elements is to initiate the Kalman filter by a very large covariance matrix; see e.g., Harvey and Phillips (1979) and Nelson and Kim (1999). While this device is computationally convenient for approximating exploratory work, it is theoretically unsatisfactory. It might lead to large rounding errors and suffer from the potential "divergence" problem, cf. Kalman and Bucy (1961) and Fitzgerald (1971). We thus follow Koopman (1997) to develop an exact initial treatment.

In model (3), the initial state vector $\boldsymbol{\gamma}_1$ can be specified as

$$\boldsymbol{\gamma}_1 = \boldsymbol{A}_0\boldsymbol{\delta} + \boldsymbol{R}_0\boldsymbol{\eta}_0,$$

where $\boldsymbol{\delta} \sim \boldsymbol{N}(\mathbf{0}, \kappa\boldsymbol{I})$ is a 2×1 random vector with $\kappa \to \infty$,

$$\boldsymbol{A}_0 = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{(r+2)\times 2}, \quad \boldsymbol{R}_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(r+2)\times r},$$

$\boldsymbol{\eta}_0 \sim \boldsymbol{N}(\mathbf{0}, \boldsymbol{Q}_0)$ is a $r \times 1$ random vector with $\boldsymbol{Q}_0$ being the unconditional variance matrix of the first r elements of $\boldsymbol{\gamma}_t$, and $\mathrm{cov}(\boldsymbol{\delta}, \boldsymbol{\eta}_0) = \mathbf{0}$. The initial conditions for the state vector become $\mathbb{E}(\boldsymbol{\gamma}_1) = \mathbf{0}$ and $\mathrm{var}(\boldsymbol{\gamma}_1) = \boldsymbol{P}$, where

$$\boldsymbol{P} = \kappa\boldsymbol{P}_\infty + \boldsymbol{P}_*$$

as $\kappa \to \infty$, $\boldsymbol{P}_\infty = \boldsymbol{A}_0\boldsymbol{A}_0'$ and $\boldsymbol{P}_* = \boldsymbol{R}_0\boldsymbol{Q}_0\boldsymbol{R}_0'$. Let $\boldsymbol{P}_{t|t-1}$ denote the covariance matrix of $\boldsymbol{\gamma}_t$ conditional on the information available up to time $t-1$. The term $\boldsymbol{P}_{t|t-1}$ can be decomposed in a similar way to matrix $\boldsymbol{P}$, i.e.,

$$\boldsymbol{P}_{t|t-1} = \kappa\boldsymbol{P}_{\infty,t|t-1} + \boldsymbol{P}_{*,t|t-1} + O(\kappa^{-1}), \quad t = 1, 2, 3, \ldots, 3T,$$

[3]A state is said to be diffuse if its covariance matrix is arbitrarily large. Diffuse initial states arise in the context of model nonstationarity.

where $\boldsymbol{P}_{\infty,t|t-1}$ and $\boldsymbol{P}_{*,t|t-1}$ do not depend on κ. It has been shown by Koopman (1997) that the influence of the term $\boldsymbol{P}_{\infty,t|t-1}$ will disappear after a limited number of updates n of the exact initial Kalman filter. Therefore, the standard Kalman filter algorithm can be applied to evaluate the model (3) for $t = n+1, n+2, \ldots, 3T$ when $\kappa \to \infty$. A detailed description of this algorithm is given in the Appendix A. In Appendix B, we briefly discuss the exact initial state algorithm for $t = 1, \ldots, n$. The diffuse log-likelihood of the proposed model is thus a byproduct of the algorithms discussed in the Appendixes A and B; for a comprehensive review of the algorithms for diffuse state-space models we refer to Koopman (1997) and Koopman and Durbin (2003).

From the recursions of the exact Kalman filter algorithm we obtain the filtered series $\boldsymbol{\gamma}_{t|t} = \mathbb{E}(\boldsymbol{\gamma}_t \mid \boldsymbol{\Omega}^t)$ and all the quantities that are necessary for the evaluation of the diffuse log-likelihood function discussed in Koopman (1997), where $\boldsymbol{\Omega}^t$ is the collection of all the observed variables up to time t. The approximate quasi-maximum likelihood estimates (QMLE),

$$\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\beta}}, \hat{\psi}_1, \ldots, \hat{\psi}_p, \hat{\varphi}_1, \ldots, \hat{\varphi}_q, \hat{\sigma}_\varepsilon)',$$

can then be found using a numerical-search method. Our program is written in GAUSS which employs the BFGS (Broyden-Fletcher-Goldfarb-Shanno) search algorithm. Plugging $\hat{\boldsymbol{\theta}}$ into the smoothing recursions proposed by Koopman and Durbin (2003) we can obtain the estimated smoothed series $\boldsymbol{\gamma}_{t|T} = \mathbb{E}(\boldsymbol{\gamma}_t \mid \boldsymbol{\Omega}^T)$, which are the optimal forecasts of $\boldsymbol{\gamma}_t$ based on all information in the sample. Our estimates of the seasonally adjusted monthly GDP are then constructed by the smoothed series $\boldsymbol{\gamma}_{t|T}$,

3 Empirical Study

To demonstrate the applicability of the state-space approach, we apply the model (3) with a diffuse initial state to Taiwan's real GDP form 1961:I – 2006:II. According to the economic intuition and the quality of the data, a constant term and the change of industrial production index (IPI) are selected

Table 1: Quasi-maximum likelihood estimates of the proposed state-space model.

Estimate	Estimate	Standard error	t-statistic
$\hat{\beta}_0$	17.1668	4.5819	3.7466*
$\hat{\beta}_1$	1.6227	0.4231	3.8352*
$\hat{\psi}_1$	0.9462	0.0143	66.1678*
$\hat{\varphi}_1$	−2011.1024	200.1948	−10.0457*
$\hat{\varphi}_2$	2146.5556	211.0155	10.1725*
$\hat{\sigma}_\varepsilon$	1.6308	0.1730	9.4265*
Log-Likelihood=−3667.960	AIC=7351.9209	SIC=7377.5089	

Note: t-statistics with an asterisk are significant at the 5% level.

as observed related monthly indicators $\boldsymbol{x}_t$. These datasets are taken from the CEPD. We estimate an array of the proposed models with $0 \leq, p, q, \leq 4$. The parameters are estimated using the algorithm described in the previous section and the Appendixes. This algorithm is initialized by a broad range of random initial values. The covariance matrix of $\hat{\boldsymbol{\theta}}$ is $-\boldsymbol{H}(\hat{\boldsymbol{\theta}})^{-1}$, where $\boldsymbol{H}(\hat{\boldsymbol{\theta}})$ is the Hessian matrix of the log-likelihood function evaluated at the QMLE $\hat{\boldsymbol{\theta}}$. Among all the models considered, the Schwartz Bayesian information criterion (SIC) select the $p = 1$ and $q = 2$ model. The estimation results are summarized in Table 1. As the table shows, all parameter estimates are statistically significant at the 5% level. From Table 1, we found that the difference of IPI has a significant impact on z_t^* ($\hat{\beta}_1 = 1.6227$). We also found that the invertibility conditions for the MA part of z_t^* are obviously violated. Nevertheless, the exact finite-sample forecasts and Kalman filter algorithm are still valid regardless of whether the moving average parameters satisfy the invertibility conditions; see e.g., Hamilton (1994), p.387.

We now conduct some diagnostic checks on the estimated model, including the Ljung-Box (1978) Q test and the LM test of Engle (1982) on the ARCH effect. The relevance of diagnostic checking is usually neglected in the liter-

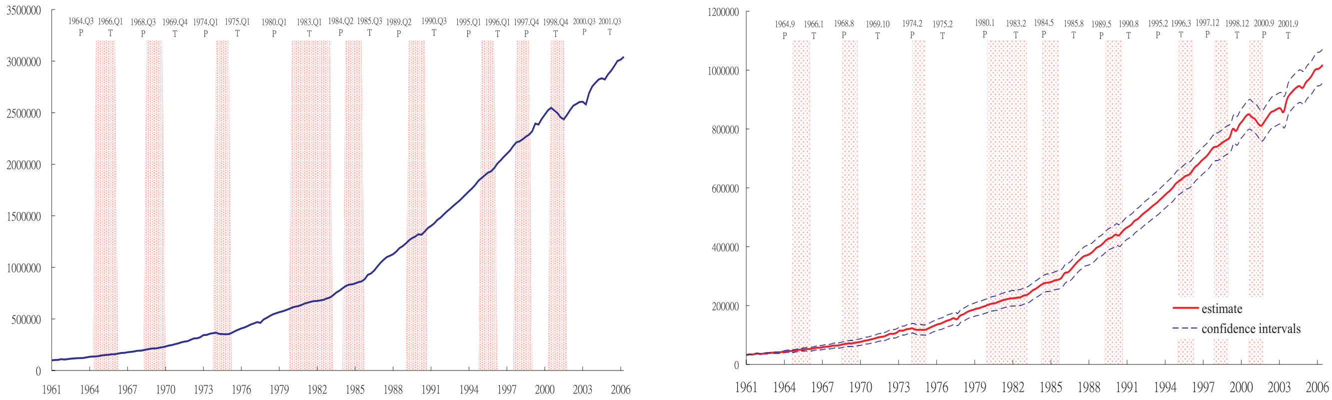

Figure 1: Published quarterly real GDP (left) and estimated monthly real GDP (right).

ature, one reason being that the innovations are not automatically available from the implementation of these methods in a classical framework. However, the smoothed mean of the residual $\hat{\varepsilon}_t = \mathbb{E}(\varepsilon_t \mid \mathbf{\Omega}^T)$ is easy to calculate and is a byproduct of the Kalman filtering algorithm; see Appendix A for more details. The resulting statistics for the residuals are $Q(12) = 18.147$, $Q(24) = 26.135$ and ARCH(4) $= 1.190$. These statistics are all insignificant at 5% level under the $\chi^2(12)$, $\chi^2(24)$ and $\chi^2(4)$ distributions. Hence, there appears no serial correlation and conditional heteroskedasticity in these residuals.

In Figure 1, we plot the published quarterly GDP and the estimated monthly real GDP in, respectively, the left and right figures. The shaded areas signify the recession periods identified by CEPD and the label "P" ("T") denotes the peak (trough). We find that both of the series in Figure 1 share a similar dynamic pattern. This shows that the estimated monthly GDP are consistent with quarterly data. In Table 2, we summarize the business cycle turning points identified by Bry and Boschan (1971) approach, using the estimated monthly real GDP, IPI, Manufacturing Sales and Nonagricultural Employment. For comparison purpose, we also check the dates of turning points by using the quarterly GDP, IPI, Manufacturing Sales and Nonagricul-

Table 2: Estimated business cycle turning points.

Peaks		Troughs	
Dataset I	Dataset II	Dataset I	Dataset II
1980:07	1980: 07	1982:07	1982:07
1984:05	1984:05	1985:09	1985:09
1987:07	1987:07	1988:04	1988:04
1989:04	1989:04	1991:02	1991:02
1991:10	1991:10	1993:12	1993:12
1995:01	1995:01	1996:08	1996:08
1998:01	1998:01	1998:12	1998:11
2000:08	2000:08	2001:08	2001:08
2002:05	2002:05	2003:04	2003:04
2004:05	2004:05	2004:11	2004:11
2005:12	2005:12	–	–

Note: Dataset I (II) includes estimated monthly real GDP (quarterly real GDP), IPI, Manufacturing Sales and Nonagricultural Employment.

tural Employment. It can be seen that both of the turning-point dates match very closely, except only for the trough on 1998. To examine the data more carefully we report the monthly disaggregation of real GDP and its forecasts in Tables 3 and 4. The estimated future values of monthly GDP from 2006:07–2006:12 are: 1021715.23, 1025819.70, 1029942.43, 1034082.42, 1038238.75, and 1042410.54, respectively. These show that the predicted quarterly GDP for the period of 2006:III to 2006:IV are 3077477.36 and 3114731.71. Such forecasts can provide us the early estimates of quarterly GDP in real time.

To demonstrate the applicability of the proposed model, we also report the estimated monthly real GDP in which the change of IPI and the change of Sales Index of Trade and Eating-Drinking places are selected as observed related monthly indicators; see Table 5 for more details. In addition, we also construct an “pseudo” annual GDP on the basis of the aggregation of the published quarterly GDP. Then, by applying the estimation algorithm proposed in this paper, we obtain “pseudo” estimated quarterly GDP. Table 6 compares the results of the “pseudo” and the published estimated quarterly GDP.

4 Conclusions

This project is concerned with the temporal disaggregation of Taiwan’s real GDP that is available only at the quarterly frequency of observations; the resulting monthly estimates incorporated with the information contained in the monthly IPI are calculated via the state-space approach. The state-space representation proposed here has several interesting features. First, it allows the use of dynamic models for the disaggregation of time series. Second, it admits high frequency related series and relaxes the assumption of co-integration relation. Third, it nests the traditional linear disaggregation techniques within more general dynamic specifications. Fourth, it computes the optimal estimates of the latent monthly GDP and provides exact finite-sample forecasts. Thus, the method proposed here adds more flexibility to the approaches introduced in the literature.

The application of the proposed model to Taiwan’s real GDP suggests that it is a useful analytical tool in calculating the monthly disaggregation of real GDP. In particular, our empirical result shows that both the estimated monthly GDP and the quarter data share a similar dynamic pattern during the analysis period. It also shows that the turning-point periods of the monthly GDP closely match the turning-point periods using the quarterly data. The proposed model thus may serve as an alternative for temporal disaggregation of time series.

With slight modifications, the methodology adopted here is flexible enough to allow for almost any kind of disaggregation problem (e.g., annual to quarterly, annual to monthly, ...) and to face interpolation, distribution and extrapolation of time series. Further, it would be very interesting to allow for certain nonlinearity system (e.g., structural change, regime switching mechanism, ...) and to concern the seasonality in the model. This is a future research direction. Research on several of the topics is currently in progress.

Appendix A: Kalman Filter

The object of the Kalman filter is to update our knowledge of the system each time a new observation y_t is brought in. In this appendix, we show the iteration and updating steps of the Kalman filter. We first denote the expectation on the variable $\boldsymbol{X}_t$, conditional on the information available up to time s, as $\boldsymbol{X}_{t|s}$. For example, conditional on the information up to time $t-1$, the mean and variance of the state vector $\boldsymbol{\gamma}_t$ are

$$\gamma_{t|t-1} = \mathbb{E}(\boldsymbol{\gamma}_t \mid \boldsymbol{\Omega}^{t-1})$$

and

$$\boldsymbol{P}_{t|t-1} = \mathbb{E}\big[(\boldsymbol{\gamma}_t - \gamma_{t|t-1})(\boldsymbol{\gamma}_t - \gamma_{t|t-1})' \mid \boldsymbol{\Omega}^{t-1}\big],$$

where $\boldsymbol{\Omega}^t - 1$ denotes the collection of all the observed variables up to time $t-1$.

We now show how to calculate $\boldsymbol{\gamma}_{t+1|t}$ and $\boldsymbol{P}_{t+1|t}$ from $\gamma_{t|t-1}$ and $\boldsymbol{P}_{t|t-1}$ recursively in model (3). Let the Kalman filter residual $\eta_{t|t-1} = y_t - \mathbb{E}(y_t \mid \boldsymbol{\Omega}^{t-1}) = \boldsymbol{h}_t'\gamma_{t|t-1}$ and its variance $f_t = \boldsymbol{h}_t'\boldsymbol{P}_{t|t-1}\boldsymbol{h}_t$ be the one-step-ahead forecast error and the one-step-ahead forecast error variance of y_t given $\boldsymbol{\Omega}^{t-1}$. Under the normality assumption of ε_t, the expectation and variance of $\boldsymbol{\gamma}_{t+1}$, conditional on $\boldsymbol{\Omega}^t$, are given by standard formulae for multivariate normal regression theory:

$$\begin{aligned} \gamma_{t+1|t} &= \boldsymbol{\mu}_t + \boldsymbol{F}\gamma_{t|t-1} + \boldsymbol{K}_t\eta_{t|t-1}, \\ \boldsymbol{P}_{t+1|t} &= \boldsymbol{F}\boldsymbol{P}_{t|t-1}\boldsymbol{L}_t' + \sigma_\varepsilon^2\boldsymbol{R}\boldsymbol{R}', \end{aligned} \tag{4}$$

for $t = 1, 2, 3, \ldots, 3T$, where

$$\begin{aligned} \boldsymbol{K}_t &= \boldsymbol{F}\boldsymbol{P}_{t|t-1}\boldsymbol{h}_t f_t^{-1}, \\ \boldsymbol{L}_t &= \boldsymbol{F} - \boldsymbol{K}_t\boldsymbol{h}_t'. \end{aligned} \tag{5}$$

Thus, with the initial values $\boldsymbol{\gamma}_{n+1|n}, \boldsymbol{P}_{n+1|n}$, we can iterate (4) – (5) to obtain all the quantities needed for computing the second part of the log-likelihood function

$$\log \mathcal{L}_2 = -\frac{3T-n}{2}\log 2\pi - \frac{1}{2}\sum_{t=n+1}^{3T}(\log|f_t| + \eta'_{t|t-1}f_t^{-1}\eta_{t|t-1}),$$

and the smoothing estimates

$$\boldsymbol{\gamma}_{t|T} = \boldsymbol{\gamma}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{s}_{t-1}, \quad t = 3T, 3T-1, \ldots, n+1,$$

where $\boldsymbol{s}_{t-1} = \boldsymbol{h}_t f_t^{-1}\eta_{t|t-1} + \boldsymbol{L}'_t\boldsymbol{s}_t$ and $\boldsymbol{s}_{3T} = \boldsymbol{0}$. The smoothed mean of residual can then be calculated by $\hat{\varepsilon}_t = \sigma_\varepsilon^2\boldsymbol{R}'\boldsymbol{s}_t$.

Appendix B: Diffuse State Filtering

Given the initializations $\boldsymbol{P}_{\infty,1|0} = \boldsymbol{P}_\infty = \boldsymbol{A}_0\boldsymbol{A}'_0$ and $\boldsymbol{P}_{*,1|0} = \boldsymbol{P}_* = \boldsymbol{R}_0\boldsymbol{Q}_0\boldsymbol{R}'_0$, the exact initial state filtering equation for (3) consists of the following equations:

$$\begin{aligned}
f_{\infty,t} &= \boldsymbol{h}'_t\boldsymbol{P}_{\infty,t|t-1}\boldsymbol{h}_t, \\
\boldsymbol{K}_{\infty,t} &= \boldsymbol{F}\boldsymbol{P}_{\infty,t|t-1}\boldsymbol{h}_t f_{\infty,t}^{-1}, \\
\boldsymbol{L}_{\infty,t} &= \boldsymbol{F} - \boldsymbol{K}_{\infty,t}\boldsymbol{h}'_t, \\
f_{*,t} &= \boldsymbol{h}'_t\boldsymbol{P}_{*,t|t-1}\boldsymbol{h}_t + \sigma_\varepsilon^2, \\
\boldsymbol{K}_{*,t} &= \big(\boldsymbol{F}\boldsymbol{P}_{*,t|t-1}\boldsymbol{h}_t - \boldsymbol{K}_{\infty,t}f_{*,t}\big)f_{\infty,t}^{-1}, \\
\boldsymbol{\gamma}_{t+1|t} &= \boldsymbol{\mu}_t + \boldsymbol{F}\boldsymbol{\gamma}_{t|t-1} + \boldsymbol{K}_{\infty,t}\eta_{t|t-1}, \\
\boldsymbol{P}_{\infty,t+1|t} &= \boldsymbol{F}\boldsymbol{P}_{\infty,t|t-1}\boldsymbol{L}'_{\infty,t}, \\
\boldsymbol{P}_{*,t+1|t} &= \boldsymbol{F}\boldsymbol{P}_{*,t|t-1}\boldsymbol{L}'_{\infty,t} - \boldsymbol{K}_{\infty,t}f_{\infty,t}\boldsymbol{K}'_{*,t} + \sigma_\varepsilon^2\boldsymbol{R}\boldsymbol{R}',
\end{aligned}$$

for $t = 1, \ldots, n$. The first part of the log-likelihood function is given by

$$\log \mathcal{L}_1 = -\frac{n}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{n}\omega_t,$$

where

$$\omega_t = \begin{cases} \log |f_{\infty,t}|, & \text{if } f_{\infty,t} \text{ is positive definite,} \\ \log |f_{*,t}| + y_t' f_{*,t}^{-1} y_t, & \text{if } f_{\infty,t} = 0. \end{cases}$$

Putting $\log \mathcal{L}_1$ and $\log \mathcal{L}_2$ together, we obtain the diffuse log-likelihood as

$$\log \mathcal{L} = \log \mathcal{L}_1 + \log \mathcal{L}_2.$$

The exact initial state smoothing equations are give by

$$\boldsymbol{\gamma}_{t|T} = \boldsymbol{\gamma}_{t|t-1} + \boldsymbol{P}_{*,t|t-1} \boldsymbol{s}_{t-1}^{(0)} + \boldsymbol{P}_{\infty,t|t-1} \boldsymbol{s}_{t-1}^{(1)},$$

where $\boldsymbol{s}_{t-1}^{(0)} = \boldsymbol{L}_{\infty,t}' \boldsymbol{s}_t^{(0)}$ and $\boldsymbol{s}_{t-1}^{(1)} = \boldsymbol{h}_t (f_{\infty,t}^{-1} \eta_{t|t-1} - \boldsymbol{K}_{*,t}' \boldsymbol{s}_t^{(0)}) + \boldsymbol{L}_{\infty,t}' \boldsymbol{s}_t^{(1)}$.

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Table 3: Monthly disaggregation of real GDP for Taiwan (I).

Date	Estimate	Date	Estimate	Date	Estimate	Date	Estimate	Date	Estimate
1961:01	32191.28	1966:01	52253.63	1971:01	86542.74	1976:01	134852.15	1981:01	214857.99
:02	32510.76	:02	52429.78	:02	87440.97	:02	136177.29	:02	216236.10
:03	33917.75	:03	52989.40	:03	88614.12	:03	137219.77	:03	217441.57
:04	33912.00	:04	53933.60	:04	90063.01	:04	137979.67	:04	218474.74
:05	33886.37	:05	54784.47	:05	91219.87	:05	139044.70	:05	219434.50
:06	33853.65	:06	55541.81	:06	92084.62	:06	140413.33	:06	220319.49
:07	33826.22	:07	56205.29	:07	92657.11	:07	142086.32	:07	221130.20
:08	34260.34	:08	56614.81	:08	93182.52	:08	143807.49	:08	221993.34
:09	35166.51	:09	56770.90	:09	93661.63	:09	145575.04	:09	222909.75
:10	36554.69	:10	56673.61	:10	94093.49	:10	147390.07	:10	223879.54
:11	37093.22	:11	56925.44	:11	94898.43	:11	148853.58	:11	224474.70
:12	36791.27	:12	57526.03	:12	96076.48	:12	149965.74	:12	224694.36
1962:01	35657.48	1967:01	58476.02	1972:01	97627.70	1977:01	150726.79	1982:01	224544.04
:02	35135.19	:02	59207.61	:02	99296.94	:02	152047.31	:02	224729.32
:03	35232.26	:03	59720.66	:03	101083.57	:03	153926.97	:03	225256.49
:04	35955.84	:04	60015.25	:04	102986.84	:04	156366.82	:04	226126.05
:05	36593.39	:05	60263.04	:05	104064.57	:05	157040.16	:05	226751.05
:06	37150.51	:06	60464.75	:06	104317.09	:06	155947.49	:06	227131.81
:07	37632.86	:07	60620.51	:07	103744.10	:07	153089.24	:07	227268.47
:08	38038.71	:08	61130.78	:08	103779.36	:08	153001.05	:08	228069.07
:09	38373.08	:09	61995.43	:09	104425.39	:09	155682.85	:09	229532.42
:10	38640.84	:10	63214.91	:10	105681.00	:10	161138.05	:10	231663.07
:11	38920.18	:11	63927.38	:11	107535.58	:11	165102.89	:11	233234.01
:12	39215.05	:12	64133.13	:12	109987.13	:12	167576.31	:12	234245.50
1963:01	39529.44	1968:01	63832.60	1973:01	113034.85	1978:01	168556.86	1983:01	234699.38
:02	39739.95	:02	63814.38	:02	114798.96	:02	169985.76	:02	235834.45
:03	39849.90	:03	64078.85	:03	115280.07	:03	171861.26	:03	237650.68
:04	39862.70	:04	64626.39	:04	114477.33	:04	174182.49	:04	240150.10
:05	39899.77	:05	65319.20	:05	114588.09	:05	176330.54	:05	242908.21
:06	39964.09	:06	66157.16	:06	115612.99	:06	178305.30	:06	245924.31
:07	40058.28	:07	67140.27	:07	117552.97	:07	180106.93	:07	249199.46
:08	40393.60	:08	67968.87	:08	118943.18	:08	181635.28	:08	251938.02
:09	40972.22	:09	68642.70	:09	119783.13	:09	182891.34	:09	254140.03
:10	41795.81	:10	69161.80	:10	120072.83	:10	183874.33	:10	255805.77
:11	42545.54	:11	69731.98	:11	120626.45	:11	184979.14	:11	257823.66
:12	43223.03	:12	70353.36	:12	121443.94	:12	186208.30	:12	260193.99
1964:01	43829.64	1969:01	71025.97	1974:01	122525.17	1979:01	187558.80	1984:01	262913.54
:02	44340.81	:02	71379.00	:02	122528.99	:02	188666.55	:02	265575.92
:03	44758.14	:03	71413.10	:03	121456.92	:03	189531.75	:03	268177.83
:04	45083.11	:04	71128.56	:04	119308.60	:04	190154.76	:04	270718.87
:05	45311.55	:05	71301.43	:05	117945.21	:05	191027.39	:05	272913.54
:06	45445.00	:06	71931.73	:06	117368.23	:06	192151.24	:06	274763.19
:07	45484.78	:07	73019.97	:07	117578.73	:07	193525.87	:07	276267.25
:08	45634.72	:08	73857.47	:08	117666.93	:08	194849.10	:08	277263.48
:09	45896.22	:09	74444.33	:09	117633.65	:09	196120.70	:09	277749.86
:10	46270.37	:10	74780.04	:10	117478.27	:10	197340.89	:10	277726.59
:11	46817.11	:11	75230.10	:11	117329.66	:11	198669.01	:11	278114.67
:12	47537.07	:12	75795.06	:12	117188.36	:12	200106.29	:12	278917.33
1965:01	48430.62	1970:01	76474.92	1975:01	117055.04	1980:01	201651.24	1985:01	280135.57
:02	49140.57	:02	77342.41	:02	117495.52	:02	203058.03	:02	281457.84
:03	49667.48	:03	78397.44	:03	118509.56	:03	204328.86	:03	282886.69
:04	50011.56	:04	79639.73	:04	120098.62	:04	205459.72	:04	284423.25
:05	50291.25	:05	80571.88	:05	121812.23	:05	206318.98	:05	285594.00
:06	50506.93	:06	81194.20	:06	123650.68	:06	206907.56	:06	286398.98
:07	50659.71	:07	81506.87	:07	125613.39	:07	207225.83	:07	286839.43
:08	50967.31	:08	82102.16	:08	127373.26	:08	207906.08	:08	287775.74
:09	51430.21	:09	82978.91	:09	128929.59	:09	208948.58	:09	289206.76
:10	52049.46	:10	84137.85	:10	130282.20	:10	210352.57	:10	291134.81
:11	52392.54	:11	85118.54	:11	131720.21	:11	211806.16	:11	294518.15
:12	52461.38	:12	85920.07	:12	133243.56	:12	213308.08	:12	299355.29

Table 4: Monthly disaggregation of real GDP for Taiwan (II).

Date	Estimate	Date	Estimate	Date	Estimate	Date	Estimate	Date	Estimate
1986:01	305648.51	1991:01	465178.13	1996:01	649831.49	2001:01	836090.43	2006:01	1003251.99
:02	309832.03	:02	467311.48	:02	654139.53	:02	832986.34	:02	1004282.29
:03	311905.04	:03	469558.49	:03	659065.28	:03	828469.05	:03	1006550.95
:04	311867.65	:04	471920.63	:04	664608.26	:04	822543.15	:04	1010057.79
:05	313042.38	:05	475076.32	:05	669236.81	:05	817619.49	:05	1013750.97
:06	315430.65	:06	479026.79	:06	672949.86	:06	813704.45	:06	1017630.04
:07	319033.60	:07	483773.60	:07	675749.32	:07	810800.66	:07	1021715.23*
:08	322927.47	:08	487426.35	:08	678971.29	:08	810898.17	:08	1025819.70*
:09	327111.62	:09	489986.97	:09	682614.77	:09	814002.84	:09	1029942.43*
:10	331585.00	:10	491457.72	:10	686681.59	:10	820110.18	:10	1034082.42*
:11	335938.07	:11	493727.58	:11	690475.60	:11	825860.97	:11	1038238.75*
:12	340169.04	:12	496796.06	:12	693998.33	:12	831249.97	:12	1042410.54*
1987:01	344275.45	1992:01	500662.38	1997:01	697253.06	2002:01	836272.02		
:02	348059.57	:02	504325.98	:02	700469.67	:02	841481.24		
:03	351520.50	:03	507781.65	:03	703649.49	:03	846880.88		
:04	354655.26	:04	511031.83	:04	706793.89	:04	852470.39		
:05	357878.80	:05	514062.75	:05	710648.18	:05	856535.61		
:06	361189.83	:06	516875.73	:06	715212.62	:06	859077.44		
:07	364587.47	:07	519468.59	:07	720482.23	:07	860092.37		
:08	367126.50	:08	522266.05	:08	725519.01	:08	861566.49		
:09	368808.37	:09	525269.11	:09	730320.26	:09	863498.18		
:10	369632.36	:10	528474.39	:10	734885.53	:10	865887.36		
:11	370743.22	:11	531608.84	:11	737910.59	:11	867978.64		
:12	372141.21	:12	534669.74	:12	739395.11	:12	869773.49		
1988:01	373828.47	1993:01	537660.08	1998:01	739344.73	2003:01	871275.59		
:02	375887.66	:02	540526.19	:02	740178.28	:02	869611.55		
:03	378320.89	:03	543270.31	:03	741902.42	:03	864788.94		
:04	381125.73	:04	545890.69	:04	744514.53	:04	856811.40		
:05	384417.75	:05	548829.77	:05	747268.80	:05	857425.89		
:06	388195.97	:06	552088.08	:06	750163.62	:06	866627.02		
:07	392459.49	:07	555664.28	:07	753200.91	:07	884417.52		
:08	395778.68	:08	559141.17	:08	755918.39	:08	898104.61		
:09	398151.85	:09	562517.53	:09	758317.56	:09	907679.95		
:10	399579.02	:10	565793.20	:10	760403.07	:10	913135.66		
:11	401509.46	:11	569159.65	:11	762403.67	:11	918030.90		
:12	403943.04	:12	572613.24	:12	764320.84	:12	922362.68		
1989:01	406881.25	1994:01	576159.59	1999:01	766153.63	2004:01	926137.83		
:02	410134.67	:02	579482.63	:02	771682.91	:02	929897.86		
:03	413702.38	:03	582584.04	:03	780904.34	:03	933636.22		
:04	417584.05	:04	585464.59	:04	793818.16	:04	937353.08		
:05	420929.78	:05	588585.87	:05	800374.93	:05	940543.67		
:06	423737.20	:06	591945.51	:06	800576.13	:06	943214.02		
:07	426005.65	:07	595545.91	:07	794417.76	:07	945363.86		
:08	427903.51	:08	599671.73	:08	793274.94	:08	945442.92		
:09	429432.95	:09	604317.24	:09	797147.72	:09	943452.36		
:10	430591.42	:10	609485.76	:10	806038.21	:10	939394.80		
:11	432604.67	:11	613792.25	:11	813228.68	:11	939378.34		
:12	435473.10	:12	617238.25	:12	818719.00	:12	943403.40		
1990:01	439196.16	1995:01	619821.36	2000:01	822503.41	2005:01	951469.24		
:02	440871.08	:02	622423.00	:02	826812.48	:02	957795.53		
:03	440498.00	:03	625041.89	:03	831639.63	:03	962382.25		
:04	438080.77	:04	627678.14	:04	836983.15	:04	965227.93		
:05	437975.70	:05	630585.55	:05	841650.56	:05	968836.24		
:06	440182.36	:06	633768.23	:06	845639.29	:06	973205.34		
:07	444703.93	:07	637222.38	:07	848953.14	:07	978337.23		
:08	449042.78	:08	639722.32	:08	849739.09	:08	983999.38		
:09	453196.36	:09	641269.08	:09	847997.68	:09	990197.41		
:10	457165.53	:10	641862.39	:10	843733.27	:10	996931.93		
:11	460484.85	:11	643487.08	:11	840326.69	:11	1001352.29		
:12	463156.29	:12	646143.09	:12	837777.34	:12	1003459.22		

Note: Data with an asterisk are predicted monthly real GDP.

Table 5: Monthly disaggregation of real GDP for Taiwan: Two related variables.

Date	Estimate	Date	Estimate	Date	Estimate	Date	Estimate
1999:01	770598.8216	2001:01	836680.0981	2003:01	871035.4068	2005:01	953033.2718
:02	773023.3089	:02	832085.0791	:02	866647.3698	:02	956080.8798
:03	775118.7495	:03	828780.6528	:03	867993.3034	:03	962532.8585
:04	795498.0073	:04	821723.9018	:04	854620.4743	:04	964327.2364
:05	795664.6237	:05	817628.1186	:05	862158.5948	:05	969430.0663
:06	803606.5890	:06	814515.0696	:06	864085.2410	:06	973512.2073
:07	791630.8323	:07	810572.0718	:07	887679.0740	:07	978837.0033
:08	797005.0092	:08	811881.4126	:08	894969.6272	:08	984373.6427
:09	796204.5885	:09	813248.1856	:09	907553.3888	:09	989323.3740
:10	807887.7452	:10	820767.9162	:10	911912.7958	:10	997364.7044
:11	811321.1423	:11	825261.2459	:11	918602.0874	:11	1000132.558
:12	818777.0025	:12	831191.9579	:12	923014.3568	:12	1004246.177
2000:01	821791.5386	2002:01	836140.9385	2004:01	926089.3182	2006:01	1002303.929
:02	827625.8945	:02	841869.2546	:02	930200.4188	:02	1005219.248
:03	831538.0869	:03	846623.9469	:03	933382.1730	:03	1006562.063
:04	837629.1643	:04	852699.7200	:04	937845.8653	:04	1010601.384
:05	841786.6419	:05	855839.0658	:05	940571.6219	:05	1013551.221
:06	844857.1838	:06	859544.6542	:06	942693.2828	:06	1017286.195
:07	849172.3225	:07	859428.9518	:07	945191.1739	:07	
:08	848701.4895	:08	862157.0178	:08	943998.0885	:08	
:09	848816.0980	:09	863571.0704	:09	945069.8776	:09	
:10	843048.1280	:10	866682.1882	:10	938225.2896	:10	
:11	841476.3730	:11	868407.3024	:11	941681.0248	:11	
:12	837312.7990	:12	868549.9994	:12	942270.2355	:12	

Table 6: Pseudo quarterly disaggregation of real GDP for Taiwan.

Date	Published	Estimate	Date	Published	Estimate	Date	Ppublished	Estimate
1961:I	98619.79	96329.15	1976:I	408249.21	413418.77	1991:I	1402048.10	1411851.00
:II	101652.01	102758.65	:II	417437.69	414553.94	:II	1426023.75	1419746.97
:III	103253.07	102547.20	:III	431468.85	434724.57	:III	1461186.92	1464853.28
:IV	110439.18	112329.05	:IV	446209.39	440667.87	:IV	1481981.36	1474788.87
1962:I	106024.92	104140.81	1977:I	456701.07	454185.25	1992:I	1512770.01	1520963.25
:II	109699.74	115180.94	:II	469354.46	459305.96	:II	1541970.31	1532251.95
:III	114044.64	108554.31	:III	461773.14	483496.21	:III	1567003.75	1574989.21
:IV	116776.07	118669.31	:IV	493817.25	484658.49	:IV	1594752.96	1588292.62
1963:I	119119.29	113995.41	1978:I	510403.88	520357.16	1993:I	1621456.57	1628564.27
:II	119726.56	123284.35	:II	528818.33	521414.89	:II	1646808.53	1639761.89
:III	121424.10	121071.72	:III	544633.55	546001.46	:III	1677322.98	1687560.04
:IV	127564.38	129482.85	:IV	555061.76	551144.01	:IV	1707566.10	1697267.98
1964:I	132928.59	129869.89	1979:I	565757.10	565056.74	1994:I	1738226.26	1751644.95
:II	135839.66	136247.54	:II	573333.40	573447.92	:II	1765995.97	1760765.08
:III	137015.71	137229.97	:III	584495.66	585990.04	:III	1799534.88	1809894.64
:IV	140624.55	143061.11	:IV	596116.20	595207.66	:IV	1840516.26	1821968.69
1965:I	147238.66	145606.30	1980:I	609038.14	606234.27	1995:I	1867286.24	1866428.67
:II	150809.74	151208.26	:II	618686.26	616708.30	:II	1892031.93	1881330.66
:III	153057.24	152949.49	:III	624080.49	626943.57	:III	1918213.79	1922184.08
:IV	156903.38	158244.97	:IV	635466.81	637385.56	:IV	1931492.56	1939081.12
1966:I	157672.81	158923.04	1981:I	648535.66	647032.37	1996:I	1963036.30	1981177.49
:II	164259.88	164774.92	:II	658228.73	661000.42	:II	2006794.92	1994523.04
:III	169591.00	167041.45	:III	666033.30	659320.95	:III	2037335.39	2047265.06
:IV	171125.08	171909.36	:IV	673048.60	678492.55	:IV	2071155.52	2055356.55
1967:I	177404.29	176452.86	1982:I	674529.85	666101.59	1997:I	2101372.22	2119423.70
:II	180743.04	181259.12	:II	680008.91	684282.32	:II	2132654.70	2131539.87
:III	183746.71	185114.05	:III	684869.96	688360.80	:III	2176321.50	2174766.30
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1968:I	191725.83	193351.44	1983:I	708184.51	720657.65	1998:I	2221425.43	2213677.52
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1969:I	213818.07	211210.75	1984:I	796667.29	802615.90	1999:I	2318740.88	2340087.58
:II	214361.72	215639.95	:II	818395.60	808488.60	:II	2394769.22	2355518.51
:III	221321.76	222403.07	:III	831280.59	826646.08	:III	2384840.43	2414333.57
:IV	225805.20	226052.98	:IV	834758.58	843351.48	:IV	2437985.89	2426396.76
1970:I	232214.77	235391.15	1985:I	844480.10	837405.01	2000:I	2480955.52	2494536.69
:II	241405.81	238020.44	:II	856416.23	853582.09	:II	2524272.99	2525361.08
:III	246587.94	249955.45	:III	863821.93	877297.37	:III	2546689.91	2491140.68
:IV	255176.46	252017.94	:IV	885008.25	881442.04	:IV	2521837.30	2562717.26
1971:I	262597.83	265979.56	1986:I	927385.58	934389.07	2001:I	2497545.83	2424949.76
:II	273367.50	267683.77	:II	940340.68	930297.11	:II	2453867.09	2497833.91
:III	279501.26	282666.70	:III	969072.69	993893.76	:III	2435701.67	2454872.36
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1972:I	298008.20	300659.74	1987:I	1043855.53	1059929.36	2002:I	2524634.14	2539333.31
:II	311368.50	301280.65	:II	1073723.89	1053892.97	:II	2568083.44	2552167.63
:III	311948.84	321761.75	:III	1100522.34	1106288.99	:III	2585157.04	2585034.83
:IV	323203.70	320827.10	:IV	1112516.79	1110507.24	:IV	2603639.49	2604978.34
1973:I	343113.87	345020.21	1988:I	1128037.02	1141328.28	2003:I	2605676.08	2617858.92
:II	344678.41	347146.67	:II	1153739.45	1149197.16	:II	2580864.31	2636306.71
:III	356279.28	352003.16	:III	1186390.01	1189121.95	:III	2690202.09	2685306.37
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1974:I	366511.07	348284.34	1989:I	1230718.31	1243092.04	2004:I	2789671.91	2778185.75
:II	354622.04	360909.14	:II	1262251.04	1250474.85	:II	2821110.77	2784806.30
:III	352879.31	353184.83	:III	1283342.11	1283099.29	:III	2834259.14	2841264.11
:IV	351996.29	363630.40	:IV	1298669.19	1298314.47	:IV	2822176.55	2862962.21
1975:I	353060.12	361048.58	1990:I	1320565.24	1311421.33	2005:I	2871647.01	2888103.12
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:IV	395245.96	384136.46	:IV	1380806.67	1367049.75	:IV	3001743.44	2977342.52